Karunya University
(Karunya Institute of Technology and Sciences)
(Declared as Deemed to be University under Sec.3 of the UGC Act, 1965)

End Semester Examinations - 2015-16 Even Semester - May 2016

15MA3008 Partial Differential Equations

Set B | **Time : 3 hrs**
Total Marks: 100

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1. (a) Find the characteristics of the equation $pq = z$ and determine the integral surface which passes through the straight line $x = 1,\ z = y$. (12)
(b) Derive the Charpit's equations of a first order nonlinear partial differential equation $f(x, y, z, p, q) = 0$. (8)

OR

2. (a) Prove that the general solution of the linear Partial Differential Equation $Pp + Qq = R$ can be written in the form $F(u,v) = 0$ where F is an arbitrary function and $u(x, y, z) = C_1$ and $v(x, y, z) = C_2$ form a solution of the equation
$$\frac{dx}{P(x,y,z)} = \frac{dy}{Q(x,y,z)} = \frac{dz}{R(x,y,z)}. \qquad (10)$$
(b) Show that the following partial differential equations $xp - yq = x$ and $x^2p + q = xz$ are compatible and hence find their solution. (10)

3. (a) Derive the canonical form for the second order parabolic partial differential equation. (10)
(b) Classify and transform the following equation to a canonical form.
$\sin^2(x)u_{xx} + \sin(2x)u_{xy} + \cos^2(x)u_{yy} = x$ (10)

OR

4. Reduce the Tricomi equation $u_{xx} + u_{yy} = 0,\ x \neq 0$ for all x, y to canonical form. (20)

5. (a) Derive the solution of the two dimensional Laplace equation in Cartesian form using variable seperable method. (10)
(b) Solve the Dirichlet's problem for a rectangle. (10)

OR

6. (a)Derive the solution of the Laplace equation in cylindrical coordinators using variable separable method. (20)

7. (a) Derive the solution of Diffusion equation in cylindrical coordinates.
Also determine the temperature $T(r,t)$ in the infinite cylinder $0 \le r \le a$, when the initial temperature is $T(r,0) = f(r)$, and the surface $r = a$ is maintained at 0^0 temperature. (20)

OR

8. (a) The ends A and B of a rod , 10 cm in length are kept at temperature 0^0C and 100^0C until the steady state condition prevails. Suddenly the temperature at the end A is increased to 20^0C and the end B is decresed to 60^0C . Find the temperature in the rod at time t. (17)

b) If $\delta(t)$ is a continuously differentiable Dirac delta function vanishing for large t, then

Prove that $\int_{-\infty}^{\infty} f(t)\delta'(t)dt = -f'(0)$. (3)

9. A string of length L is released from rest in the position $y = f(x)$. Show that the total energy of the string is $\frac{\pi^2 T}{4L}\sum_{n=1}^{\infty} n^2 k_n^2$, where $k_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$ and T -tension in the string.

If the mid-point of a string is pulled aside through a small distance and then released, show that in the subsequent motion the fundamental mode contributes $\frac{8}{\pi^2}$ of the total energy. (20)

Wishing you All the Best
